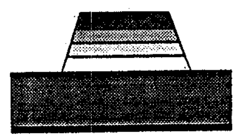
A Planar Integral Equation Method for the Analysis of Dielectric Ridge Structures Using Generalized Boundary Conditions

T.E. van Deventer, P.B. Katehi

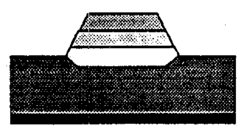
Radiation Laboratory, EECS Department, University of Michigan, Ann Arbor, MI

Abstract-

A novel method is developed to calculate the propagation characteristics of dielectric ridge structures in high frequency monolithic integrated circuits. First, the electric field in the dielectric ridge is expressed in terms of a polarization current from which an equivalent surface current density is defined. Further, generalized boundary conditions are enforced in order to provide a simple integral equation. Results derived by this modified integral equation approach give excellent agreement with other numerical methods. The main advantage of this technique is that it simplifies greatly the analysis of three-dimensional complex structures.



a) Ridged Waveguides



b) Embedded Waveguides

Figure 1: General configuration of low-loss ridged waveguides using heterostructures

1 INTRODUCTION

At the present time, almost all monolithic circuits are made of thin strip conductors which provide simplicity in the fabrication and desired guiding properties for frequencies up to the millimeter-wave region. However, this technology introduces radiation and ohmic losses which become unacceptably high as the frequency approaches the terahertz region. In order to avoid these limitations, novel dielectric guiding structures and circuit elements operating in the terahertz regime have been recently proposed that use epitaxial semiconducting materials or heterostructures on GaAs or InP substrates [1], [2]. These low-loss ridged and semi-embedded lines (Figure 1) are appropriate for high frequency monolithic applications and combine easy fabrication with good guiding properties and electrically small size. However, to design these circuit elements a rigourous theoretical characterization of the waveguiding structures is needed.

Theoretical studies on geometrically simple optical and microwave dielectric waveguides have been presented in the past decade using approximate or numerical methods. The approximate methods are represented by an analytical approximation introduced by Marcatili [3] and by the effective index method [4], [5]. The numerical methods are divided into variational methods [6], mode-matching methods, finite-element methods [7] and integral equation methods using polarization currents [8]. These methods have been exclusively applied to two-dimensional problems. Most of existing numerical techniques perform a fine discretization of the cross-section introducing many unknowns and strong numerical instabilities. Consequently, an extension of these methods to three dimensional problems introduces many practical limitations and requires special care [9].

This paper presents a two-dimensional methodology which is rather unique in terms of combined accuracy and simplicity and has demonstrated excellent performance when applied to basic dielectric structures. Furthermore, the major advantage of this technique is that it can easily be extended to three dimensional problems without increasing the complexity of the solution. In this mathematical scheme, the formulation of the electric field in terms of equivalent electric and magnetic polarization currents is shown to lead to a modified integral equation eigenproblem.

2 THEORY

For the sake of simplicity in the presentation of the technique and without loss of generality, we consider the dielectric structure shown in Figure 2 with nonmagnetic materials and with the thickness 2h equal to a fraction of the dielectric wavelength and small compared to the strip width. Under these assumptions, the material of region (3) may be replaced by an equivalent electric polarization current distribution occupying volume V_3. This current distribution is given by:

$$\vec{J}_p(\vec{r}) = j\omega(\epsilon_3 - \epsilon_0)\vec{E}_3 \quad (1)$$

where $\vec{E}_3$ is the electric field in region (3).

We now define an equivalent planar current sheet extending over the surface S_e. This current is given in terms of the electric polarization current $\vec{J}_p$ by the following relation

$$\vec{J}_s = \int_{-h}^{h} \vec{J}_p dx. \quad (2)$$

The electric field throughout volume V_3 can be written in terms of the electric field on the upper and lower interface of the surrounding surface S_3 by using a Taylor's expansion as shown below

$$\vec{E}_3(x,y) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(n+1)} \left[\frac{\partial^n \vec{E}_3(x,y)}{\partial x^n}\right]_{x=\pm h} f(x) \quad (3)$$

where $f(x)$ is given by

$$f(x) = \begin{cases} (x-h)^n & \text{if } 0 \leq x \leq h \\ (x+h)^n & \text{if } -h \leq x \leq 0 \end{cases} \quad (4)$$

In view of (3), equation (2) takes the form

$$\vec{J}_s = \vec{J}_s^+ + \vec{J}_s^- \quad (5)$$

where

$$\vec{J}_s^+ = j\omega(\epsilon_3 - \epsilon_0) \sum_{n=0}^{\infty} \frac{(-1)^{n+2} h^{n+1}}{\Gamma(n+2)} \left[\frac{\partial^n \vec{E}_3(x,y)}{\partial x^n}\right]_{x=h} \quad (6)$$

and

$$\vec{J}_s^- = j\omega(\epsilon_3 - \epsilon_0) \sum_{n=0}^{\infty} \frac{h^{n+1}}{\Gamma(n+2)} \left[\frac{\partial^n \vec{E}_3(x,y)}{\partial x^n}\right]_{x=-h} . \quad (7)$$

This equivalent surface current density radiates an electromagnetic field given by the following equation:

$$\vec{E}_r(x,y) = \int_{-w/2}^{w/2} \left[\bar{\bar{G}}(x,y/x',y')\right]_{x'=0} \cdot \vec{J}_s(y')dy' \quad (8)$$

where $\bar{\bar{G}}$ is the dyadic Green's function for the problem. If the dielectric ridge is inside a rectangular waveguide section, this function can be found analytically as a superposition of all the propagating and attenuating modes in the inhomogeneously filled waveguide (see Fig 2b). For

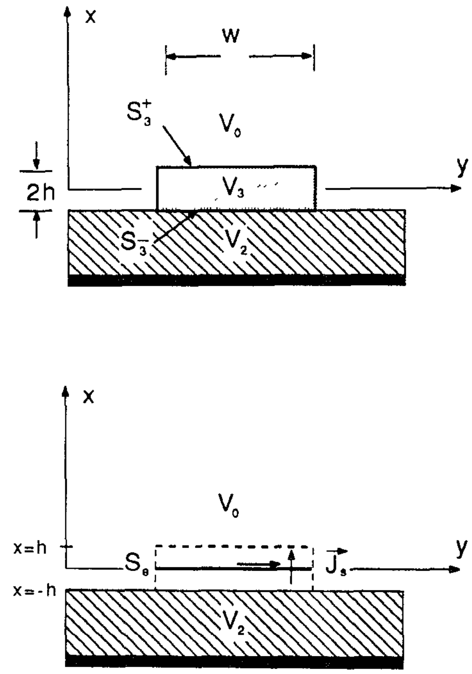


Figure 2: Equivalent polarization current

free space problems, the Green's function is written in terms of Sommerfeld integrals. In order to make the two boundary value problems presented in Figure 2 equivalent, the radiated field $\vec{E}_r$ given by equation (8) has to be identical to the original field $\vec{E}$ on the surface S_3 surrounding volume V_3. As a result, these fields satisfy the following equations:

$$\frac{\partial^n \vec{E}_r}{\partial x^n} = \frac{\partial^n \vec{E}}{\partial x^n} \;\; \forall n = 0,1,... \; on \; S_3. \quad (9)$$

On the other hand, the fields $\vec{E}$ and $\vec{E}_3$ satisfy the appropriate boundary conditions across the upper S_3^+ and lower S_3^- parts of the surface surrounding volume V_3, namely:

$$\hat{x} \times \vec{E}|_{x=\pm h} = \hat{x} \times \vec{E}_3|_{x=\pm h} \quad (10)$$
$$\epsilon\,(\hat{x} \cdot \vec{E})|_{x=\pm h} = \epsilon_3\,(\hat{x} \cdot \vec{E}_3)|_{x=\pm h}. \quad (11)$$

In addition, the y- and z- derivatives of the above fields on the air-dielectric interface are simply related through the following equations:

$$\hat{x} \times \left[\frac{\partial^n \vec{E}_3}{\partial \xi^n}\right]_{x=\pm h} = \hat{x} \times \left[\frac{\partial^n \vec{E}}{\partial \xi^n}\right]_{x=\pm h} \quad (12)$$
$$\epsilon_3 \left(\hat{x} \cdot \left[\frac{\partial^n \vec{E}_3}{\partial \xi^n}\right]_{x=\pm h}\right) = \epsilon \left(\hat{x} \cdot \left[\frac{\partial^n \vec{E}}{\partial \xi^n}\right]_{x=\pm h}\right) \quad (13)$$

where ξ denotes the y or z coordinate.

However, the relations between the derivatives of the above fields with respect to the direction vertical to the air-dielectric interface on S_3^+ and S_3^- are related through somehow more complicated expressions which can be put in the following general form:

$$\hat{x} \times \left[\frac{\partial^n \vec{E}_3}{\partial x^n}\right]_{x=\pm h} = \hat{x} \times \vec{\mathcal{F}}_{\pm}\left(\frac{\partial^m \vec{E}}{\partial x^m}{}_{x=\pm h}\right) \quad (14)$$

$$\epsilon_3\, \hat{x} \cdot \left[\frac{\partial^n \vec{E}_3}{\partial x^n}\right]_{x=\pm h} = \epsilon\, \hat{x} \cdot \vec{\mathcal{F}}_{\pm}\left(\frac{\partial^m \vec{E}}{\partial x^m}{}_{x=\pm h}\right) \quad (15)$$

$$m = 1, .., n$$

where $\vec{\mathcal{F}}_+$, $\vec{\mathcal{F}}_-$ are known analytical vector functions including higher order derivatives of $\vec{E}$ on the interfaces S_3^+ and S_3^- respectively. In equations (10) - (15), ϵ_3 is the dielectric constant of the medium in V_3, and ϵ is equal to ϵ_0 when $x = h$ and ϵ_2 when $x = -h$.

In view of the above, equation (7) takes the form

$$\frac{1}{j\omega(\epsilon_3 - \epsilon_0)}[\hat{x}\ \times, \hat{x}\ \hat{x}\ \cdot]\ \vec{J}_s = \int_{-w/2}^{w/2} \bar{\bar{G}}'(x, y/x', y')_{x'=0} \,.\, \vec{J}_s(y')dy' \quad (16)$$

where $\bar{\bar{G}}'$ is a modified Green's function given by

$$\bar{\bar{G}}'(x, y/x', y')_{x'=0} = \sum_{k_x}\sum_{k_y} \bar{\bar{g}}_{k_x k_y}(k_y; y, y')\ h(k_x x). \quad (17)$$

In equation (17), k_x and k_y are the eigenvalues along the x and y directions respectively, $h(k_x x)$ is a harmonic function of x and $\bar{\bar{g}}_{mn}(y, y')$ is a dyadic function with harmonic dependence on y, y'. This dyadic function results in simple closed-form expressions that account for the infinite summations of the derivatives. In the case of open structures, the summations of equation (17) are replaced by semi-infinite integrals. Equation (16) can be solved numerically to provide the unknown components of the equivalent surface current density. This equivalent current can then be used to derive the field distribution inside volume V_3.

3 RESULTS AND DISCUSSION

As a demonstration of the validity of the presented technique, the structure of Figure 3 has been analyzed and the propagation constant of the dominant mode has been computed as a function of the width of the dielectric strip. As predicted for the extreme cases $w = 0$ and $w = b$, the structure simplifies to a partially-filled waveguide with three (or four) homogeneous dielectric regions, for which the propagation constants are simply found by solving the appropriate characteristic equations [10], [11]. In Figure 4, the phase constant of the dominant mode has been computed as a function of frequency. In this mode, the electric field component which is parallel to the dielectric interface (E_y) is a few orders of magnitude larger than the other two components. The theoretical results of this method show very good agreement with theoretical results derived from the classical 2-D modal analysis [2]. As can be seen in Figure 4, the technique applies very efficiently even for electrically thick ridges ($w = 0.25\ \lambda_g$ at 120 GHz).

The proposed method can also be extended to layered structures by appropriately modifying the Taylor's expansion to account for the existence of the layers and the appropriate boundary conditions on the interfaces between them. Then, the solution can proceed exactly as has been previously described. As another application, this technique can be used to study the propagation characteristics and field distributions of optical waveguides, such as VLSI interconnects constructed from polyamide strips on GaAs substrates.

The planar integral equation technique can be further applied to study three-dimensional passive circuit elements such as power dividers, impedance transformers, bends and stubs. Such an extension is rather simple. With the replacement of the volume polarization current with an equivalent current of lower dimensionality, the original problem is simplified and can be treated as any other three-dimensional problem with unknown planar current densities [12]. The development of this technique allows the design of novel monolithic circuits which can provide high performance at frequencies up to the terahertz region [13].

4 CONCLUSIONS

A modified planar integral equation approach has been developed for the analysis of monolithic structures using equivalent polarization currents. Propagation characteristics are presented for a dielectric ridge on a layered substrate and compare very closely to other well-established numerical methods. The main advantage of this method lies in its simplicity when applied to the theoretical characterization of more complex 3-D circuit elements. The technique presented in this paper will be implemented to study different geometries of low-loss dielectric ridge lines and the derived theoretical results will be validated by extensive experiments.

5 ACKNOWLEDGMENTS

The authors wish to thank A.G. Engel Jr. for providing numerical results included in Figure 4. This work was supported by NASA/Center for Space Terahertz Technology at the University of Michigan.

References

[1] Linda P.B. Katehi, "High Frequency Numerical Modeling of Passive Monolithic Circuits and Antenna Feed Networks," *presented at the International*

Conference on Directions in Electromagnetic Wave Modeling, Polytechnic University of New York, New York, October 1990.

[2] A.G. Engel Jr., P.B. Katehi, "Low-loss Monolithic Transmission Lines for Sub-Mm and Terahertz Frequency Applications," *submitted to IEEE Trans. Microwave Theory Tech.*.

[3] E.A.J. Marcatili, "Slab-Coupled Waveguides," *Bell Syst. Tech. Journal*, vol. 53, pp. 645-674, 1974.

[4] G.B. Hocker, W.K. Burns, "Mode Dispersion in Diffused Channel Waveguides by the Effective Index Method," *Applied Optics*, vol. 16, pp. 113-118, 1977.

[5] C.M. Kim, B.G. Jung, C.W. Lee, "Analysis of Dielectric Rectangular Waveguide by Modified Effective-Index Method," *Electronics Letters*, vol. 22, pp. 296-298, 1986.

[6] P.K. Mishra, A. Sharma, S. Labroo, A.K. Ghatak, "Scalar Variational Analysis of Single-Mode Waveguides with Rectangular Cross-Section," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 282-286, 1985.

[7] B.M.A. Davies, J.B. Davies, "Finite-Element Solution of Integrated Optical Waveguides," *IEEE Journal of Lightwave Technology*, LT-2, pp. 682-688, 1984.

[8] T.G. Livernois, D.P. Nyquist, "Integral-Equation Formulation for Scattering by Dielectric Discontinuities Along Open-Boundary Dielectric Waveguides," *Journal of Optical Society of America*, A-4, pp. 1289-1295, July 1987.

[9] B.J. Rubin, "Full-Wave Analysis of Waveguides involving Finite-Size Dielectric Regions," *IEEE MTT-International Symposium Digest*, Dallas, pp 705-708, May 1990.

[10] R.F. Harrington, *Time-Harmonic Electromagnetics Fields*, McGraw-Hill Book Co., 1961.

[11] T.E. van Deventer, P.B. Katehi, "Computer Program for Mode Search in Partially-filled Waveguide," Radiation Laboratory Report No. 024562-3-T, The University of Michigan, Ann Arbor, MI, March 1990.

[12] W.P. Harokopus, P.B. Katehi, "Characterization of Microstrip Discontinuities on Multilayer Dielectric Substrates Including Radiation Losses," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-37, No. 12, pp. 2058-2066, December 1989.

[13] A.G. Engel Jr., T.E. van Deventer, P.B. Katehi, "Low-loss Guiding Structures for THz Frequency Applications," *submitted to the Second International Symposium on Space Terahertz Technology.*

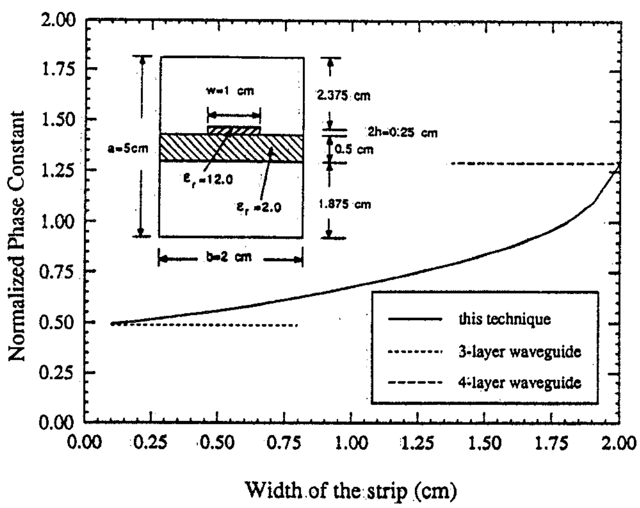


Figure 3: Phase constant as a function of strip width compared to a partially-filled waveguide structure ($\epsilon_{r1} = 1$, $\epsilon_{r2} = 2$, $\epsilon_{r3} = 12$, $h = 0.25$ cm, $f = 3$ GHz)

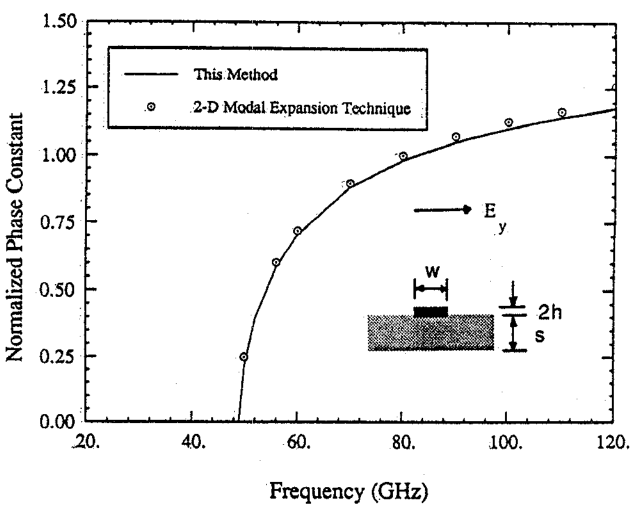


Figure 4: Comparison between the modified Green's function and the modal expansion method ($w = 0.5$ mm, $h = 62.5$ μm, $s = 250$ μm, $\epsilon_{strip} = 2$, $\epsilon_{substrate} = 12$)